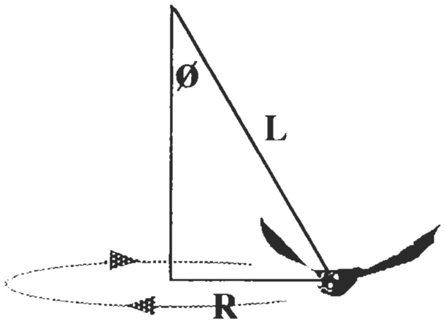
FLYING IN CIRCLES #1: WHAT DETERMINES THE ANGLE, Ø (*phi*), THE TETHERING STRING MAKES WITH VERTICAL?

GEOMETRY:
CIRCULAR FLIGHT
(Diagram 1)



L is the length of the tether.
R is the radius of the circle.

The *basic principle* behind the analysis is this: **To change the speed and/or the direction of motion of any object, you must have a non-zero net force acting on the object.** A change in speed or direction of motion of an object is called an **acceleration.** Substitute ***bat*** for ***object***, and you have our specific example.

Set up the bat according to the manufacturer's instructions. If you want to shorten the tether, you can tie a loop in it. Put the switch in the 'on' position, engaging the *bat*tery power. With the wings flapping, pull the bat out away from the center, and launch it from right to left across your body. The bat will soon settle down to a regular flight path. If not, try the launch again.

The bat is now moving at a constant speed. However, it is not moving in a straight line. It is continuously changing direction to follow its circular path. According to the *basic principle* from the first paragraph, there must be a non-zero net force acting on the bat to make it move in a circle. The bat is being accelerated.

To understand the source of this direction-changing force and its relationship to the angle, ø , let's start with the bat at rest. Stop the bat, and let the bat just hang motionless from the tether. Even though the bat is at rest, we know that there is at least one force acting on the bat. That force is induced by gravity: it is the weight of the bat and its batteries.

The weight of the bat is not zero but the net force is zero, as shown by the lack of acceleration. So, it must be that the tether supplies a force equal in size, but opposite in direction to the weight. This situation is illustrated in Diagram 2.

The arrow pointing downward represents the weight **W**). The arrow pointing upward (**T**) represents the ***tension force*** supplied by the tether.

FORCES: BAT AT REST
(Diagram 2)

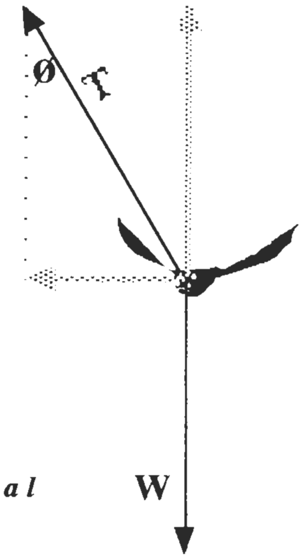
The arrows are in opposite directions, as are the forces. The arrows are of the same length, showing that the forces are of the same size. Both forces act on the bat. The two forces acting on the bat, taken together, cancel each other and leave a net force of zero, as well as an acceleration of zero.

If the tether were an elastic band, you would be able to see it stretch a little in order to apply the tension force to the bat. In fact, you can put a stretchy elastic band between the bat and the tether to get a magnified view of what the tether has to do. (A paper clip makes a good connector between the elastic band and the bat.)

This is a good time to make our first measurement. Find the length, in centimeters, of the tether. (If you tried out the elastic band, remove it first.) Record the length of the tether for future reference. ______

Restart the bat. Diagram 3 shows the forces on the bat in its circular motion.

FORCE DIAGRAM: BAT IN CIRCULAR MOTION
(Diagram 3)



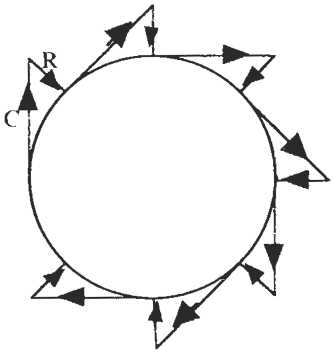
The tether is at an angle, **ø,** to the vertical, so the tension force applied by the tether is also inclined in the same way. When the bat is in circular motion, the tension force is larger than the weight, and no longer is it directly opposite in direction to the weight.

The tension force now has two jobs to perform. First, it must still provide enough vertical force to balance out the weight. This ***vertical component*** of the tension force is represented by the vertical dotted arrow. It is in the opposite direction of the weight and the same size as the weight.

Second, the ***horizontal component*** of the tension force provides a net (unbalanced) force to cause the acceleration associated with the changing direction of motion of the bat. This force is represented by the horizontal dotted arrow. Throughout the flight of the bat, the *horizontal component* of the tension force points *toward the center* of the circle. Label the tension force components on Diagram 3 for future use.

The general name used for a force toward the center of curved motion is ***centripetal force.*** The *horizontal component* of the tension force provides the *centripetal force*. (Please note: this is *centripetal force*, **not** centrifugal force. There is no force directed away from the center of motion.)

As a result of this *centripetal force*, there will be a continual change in the direction of motion, a centripetal acceleration, but no change in the speed. There will be uniform circular motion.



One way to think about uniform circular motion is to imagine the motion to be made up of two parts, one tangent to the circle (C), and one toward the center (R). This diagram shows an exaggerated version of what would happen if the circle were completed in 8 distinct steps (of which 7 are illustrated). The correction toward the center at the end of each step is the accelerated motion. One can think about the motion of the moon about the earth in a similar way, with the earth's gravity providing the centripetal force. If the effect were continuous, done in a infinite number of steps instead of just 8, the result would be a perfectly smooth motion.

The angle, ø, that the tether makes with vertical is determined by how much the tension force must be inclined to do its two jobs simultaneously. We can find out the size of that angle in the following way.

While the bat is flying along in uniform circular motion, measure (in centimeters) the radius of the circle. This may take some ingenuity. But, that's part of the fun, to figure out a way to get a good measurement while the bat is in motion. Record the radius, **R**, of the circle here _______.

Compare Diagram 3 and Diagram 1. Diagram 3 is about the size and direction of forces, and Diagram 1 is about lengths. But their geometry is the same.

There is only one right triangle that has a hypotenuse equal to the length of the tether and a base equal to the length of the radius of the circle the bat is flying. That triangle has the same value of ø as the one which lets the tension do its two jobs simultaneously.

You can construct a scale model of this triangle, which also has the same ø value, using a base and a hypotenuse 1/10 as large as your actual measurements of **L** and **R**. Triangles which are scale models of each other are called ***similar triangles***. Even though the lengths of the sides of the two triangles are different, their corresponding angles have the same measure.

We will use our measurements of **L** and **R** to illustrate a method for constructing the scale model triangle. Since we used a shortened tether due to space limitations, our values will probably be different from yours, but the process will be the same. Trust your data. We had L = 66.0 cm and R = 58.6 cm.

We used the lower left hand corner of a copy of these instructions for the right angle. From the lower left corner, we measured 5.86 cm along the bottom of the page and made a mark. Then, we broke a piece of uncooked spaghetti to a length of 6.60 cm. (really!). We put one end of the spaghetti at the mark we made on the bottom of the page. We kept it there while we pivoted that spaghetti piece so that the other end just met the left side of the page.

By measuring the angle between the spaghetti and the left side of the page with a protractor, we got ø . From our data and triangle, we got an angle of 63 degrees. What did ***you*** get with your data and triangle? ______

(Keep your spaghetti for use in part 2. *Flying in Circles #2* shows how to compute the values of the tension force and the centripetal force.)